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INTERSECTING BRANES AND SUPERSYMMETRY[1]

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ABSTRACT

We consider intersecting M-brane solutions of supergravity in eleven dimensions. Supersymmetry turns out to be a powerful tool in obtaining such solutions and their generalizations.

1. Introduction

The revival of the concept of strong-weak coupling duality has drastically changed our view of string theories. The five apparently different ten-dimensional superstring theories are now interpreted as different limits of a single theory, the conjectured M-theory. The study of extended objects, which by duality must manifest themselves in each of the descendents of M-theory, has been a decisive factor in establishing this picture of a united string theory[2].

Of particular interest are those extended objects (p-branes, where p is the dimension of the spatial extension) which satisfy a BPS-bound and preserve partial supersymmetry. Such objects can satisfy a "no-force" condition, implying that static configurations of several such objects can exist due to a cancellation of the gravitational and gauge forces between them. Several authors have contributed to the rather complete picture that now exists of these intersecting p-brane configurations [2, 3, 4, 5, 6, 7, 8]. Here I would like to report on the work done in [8], where a classification of multiple intersections in $D = 10$ and $D = 11$ was obtained. I will limit myself to our results in eleven dimensions, and, in the spirit of this meeting, I would like to discuss in particular how supersymmetry can be helpful in obtaining intersections of M-branes. In particular, we will find that supersymmetry is a useful guide in constructing the intersections of the $M2$- and $M5$-brane, and it shows that these should be extended to include objects with 1, 6, and 9 spatial extensions.

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[2]For a recent review of these developments, see, e.g., [1]

2. Pair Intersections

The basic solutions in $D=11$ are the $M2$-brane [9]:

$$ds^2 \;=\; H^{-2/3}\, dx^2_{(0-2)} - H_2^{1/3}\, dx^2_{(3-10)}, \qquad F_{012i} = \partial_i H^{-1}, \tag{1}$$

where H is harmonic on the eight-dimensional space transverse to the membrane, and the $M5$-brane solution[3]: [10]:

$$ds^2 \;=\; H^{-1/3}\, dx^2_{(0-5)} - H^{2/3}\, dx^2_{(6-10)}, \qquad F_{012345i} = \partial_i H^{-1}. \tag{2}$$

In this case H is harmonic on the five-dimensional transverse space.

For our purposes it is useful to represent the metric for these solutions pictorially as

$$ds^2 = \underbrace{\times\ \times\ \ldots\ \times}_{p+1}\ \overbrace{-\ -\ \ldots\ -}^{10-p}, \tag{3}$$

where $\times$ indicates a worldvolume coordinate of, $-$ a direction transverse to the p-brane. In this notation, the basic intersections [2, 3, 5] of the $M2$- and $M5$-brane can be represented by[4]

$$(0|M2,M2) \;=\; \left\{ \begin{array}{c|cccccccccc} \times & \times & \times & - & - & - & - & - & - & - & - \\ \times & - & - & \times & \times & - & - & - & - & - & - \end{array} \right. , \tag{4}$$

$$(1|M2,M5) \;=\; \left\{ \begin{array}{c|cccccccccc} \times & \times & \times & - & - & - & - & - & - & - & - \\ \times & \times & - & \times & \times & \times & \times & - & - & - & - \end{array} \right. , \tag{5}$$

$$(3|M5,M5) \;=\; \left\{ \begin{array}{c|cccccccccc} \times & \times & \times & \times & \times & \times & - & - & - & - & - \\ \times & \times & \times & \times & - & - & \times & \times & - & - & - \end{array} \right. , \tag{6}$$

$$(1|M5,M5) \;=\; \left\{ \begin{array}{c|cccccccccc} \times & \times & \times & \times & \times & \times & - & - & - & - & - \\ \times & \times & - & - & - & - & \times & \times & \times & \times & - \end{array} \right. . \tag{7}$$

Each intersection is determined by two harmonic functions, H_1 and H_2. We distinguish between overall worldvolume directions (both rows have an $\times$, the harmonic functions are in all cases independent of these directions), relative transverse directions (only one row has an $\times$), and overall transverse directions (both rows have a $-$). In (4-6) either both H_i must depend on the overall transverse directions, or one H must depend on overall transverse, the other on relative transverse directions. In (7) the dependence of the H_i must be on the relative transverse directions only.

The metric for these basic pairs is easily constructed. In general, in the intersection of type $(q|q+r, q+s)$ the form of the metric is

$$\begin{aligned} ds^2 \;=\; & H_1^{\alpha_1} H_2^{\alpha_2} \{ dx^2_{(0-q)} - H_1 dx^2_{(q+1,q+s)} \\ & H_2 dx^2_{(q+s+1,q+s+r)} - H_1 H_2 dx^2_{(q+r+s+1,10)} \} \,. \end{aligned} \tag{8}$$

[3]Supergravity in $D=11$ is formulated in terms of a three-form gauge field. For the solutions considered here the contribution of the Chern-Simons term to the equations of motion, which depends on the three-form gauge field, does not contribute. In that case it is possible to represent the fivebrane in terms of a six-form gauge field, the field strength $F_{012345i}$ being the dual of F_{jklm}.

[4]We denote the intersection of a p_1- and a p_2-brane over a common $q+1$ dimensional spacetime by $(q|p_1,p_2)$.

Here α is $-2/3$ for $M2$, $-1/3$ for $M5$. The curvature tensors F for the basic pairs correspond to the sum of the curvatures of the separate branes, except for (7), where a slight modification is required ([5, 4]).

The basic rule in constructing intersections of $N > 2$ fundamental objects is, that each pair among the N objects must be one of the above pairs. This leads to configurations with a maximum of nine branes [8]. In the next section, we will discuss the role of supersymmetry in obtaining multiple intersections.

3. Supersymmetry

The BPS $M2$-and $M5$-brane each preserves 1/2 of the $D = 11$ supersymmetry. The supersymmetry transformation of the gravitino reads:

$$\delta\psi_\mu = \partial_\mu\epsilon - \tfrac{1}{4}\omega_\mu{}^{ab}\epsilon - \tfrac{i}{576}\left(\Gamma_\mu\Gamma^{abcd} - 3\,\Gamma^{abcd}\Gamma_\mu\right)\epsilon\, F_{abcd}\,. \tag{9}$$

Supersymmetry is partially preserved, if the configuration is such that $\delta\psi_\mu$ vanishes for some ϵ. For $M2$ and $M5$ a simple calculation leads to the following conditions:

$$\begin{aligned} M2\ :\ \epsilon &= H^{-1/6}\eta,\ \eta \text{ constant with } P_2\eta = \eta,\ \text{ where } P_2 = i\Gamma^{012}\,, && (10)\\ M5\ :\ \epsilon &= H^{-1/12}\eta,\ \eta \text{ constant with } P_5\eta = \eta,\ \text{ where } P_5 = \Gamma^{012345}\,. && (11) \end{aligned}$$

So η is algebraically restricted by a product of Γ-matrices corresponding to the worldvolume directions.

Given the supersymmetry preserving conditions (10, 11), the obvious question is how to formulate the preservation of supersymmetry for pairs of M-branes. If η must satisfy two conditions, then compatibility requires that the corresponing P_p must *commute*. For a pair consisting of a p_1 and a p_2 brane, intersecting over a common worldvolume of dimension $d_{12}+1$, one can derive the following rule:

- If both p_1 and p_2 are even, d_{12} must be even, otherwise d_{12} must be odd.

Such a pair will preserve 1/4 of the $D = 11$ supersymmetry. For M_2 and M_5 this condition leads precisely to the four possibilities given in (4-7).

Once intersections of three or more fundamental branes have been obtained, there is a simple method to add additional branes which do not lead to further supersymmetry breaking. Consider a triple p_1, p_2 and p_3 satisfying the above conditions, i.e., such that the P_{p_i} commute. Then the product $P_{p_4} \equiv P_{p_1}P_{p_2}P_{p_3}$ clearly commutes with each P_i, and a brane with spatial extension p_4 can be added to the configuration. Note that this calculation also determines the orientation of the p_4-brane.

For any allowed triple of $M2$ and $M5$, one finds that p_4, calculated as above, is always one of the numbers $1, 2, 5, 6, 9$, i.e., p_4 is of the form $4k + 1$ or $4k + 2$. More precisely, we find the following: Let p_1, p_2 and p_3 form an intersecting triple with 1/8 supersymmetry, then

- If either one or three p_i are of the form $4k + 1$, then so is p_4, otherwise p_4 is of the form $4k + 2$.

It now becomes interesting to extend the intersecting pairs of Section 2 to the case of M-branes with spatial dimensions $1, 2, 5, 6, 9$. As we have seen above, the allowed pairs are determined

by supersymmetry. The result is given in the Table 1. In this table we have left out intersections of the form $(p|p,p)$, where the two intersecting branes overlap completely. These are still expressed in terms of a single harmonic function and preserve 1/2 of supersymmetry. In the table the numbers d_{12}, p_1 and p_2 are therefore restricted by $d_{12} < \max(p_1, p_2)$. The fact that the configuration must fit in ten spatial dimensions implies $p_1 + p_2 - d_{12} \leq 10$.

p_i	1	2	5	6	9
1	–	1	1	1	1
2	1	0	1	0, 2	1
5	1	1	1, 3	1, 3	5
6	1	0, 2	1, 3	2, 4	5
9	1	1	5	5	–

(12)

Table 1. **Basic pair intersections** $(d_{12}|p_1, p_2)$ **in** $D = 11$**.** The table indicates the possible values of d_{12} for each pair p_1 and p_2. The 2- and 5-branes are discussed in Section 2, the nature of 1-, 6- and 9-branes in Section 4.

We have seen that supersymetry determines the pair intersections, and is helpful in obtaining, for a given configuration, an additional brane which does not lead to further supersymmetry breaking. For the last point we used triple configurations with 1/8 supersymmetry. A further use of supersymmetry arises for the pair intersections themselves. Consider a pair $(d_{12}|p_1, p_2)$. By taking the product of P_{p_1} and P_{p_2} we obtain a matrix $\Gamma^{(p_1+p_2-2d_{12})}$, where (p) stands for a set of p spatial indices. The indices correspond to the relative transverse coordinates of the pair. This matrix does not involve Γ^0, so the worldvolume is spacelike and cannot be used to define an additional brane. But in $D = 11$ the matrix $i\Gamma^{012\ldots10} = 1$. Therefore $\Gamma^{(p_1+p_2-2d_{12})} = i\Gamma^{0(10-p_1-p_2+2d_{12})}$, which does define a suitable worldvolume. Note that if p_1 and p_2 are both of the form $4k+1$ or $4k+2$, then so is $10-p_1-p_2+2d_{12}$. In this way we can obtain configurations of three branes with 1/4 supersymmetry, which have no overall transverse directions. However, one has to be careful with the way the harmonic functions are allowed to depend on the coordinates. Following the rules for intersecting pairs, one finds that only in a few cases a nontrivial solution arises. There is only one example involving only $M2$ and $M5$. This arises from the pair $(1|5,5)$ (see (7)), to which we can add an $M2$, such that the triplet has a common string direction (see also [20, 21]).

4. The 1-, 6- and 9-brane

In Table 1 we find the pairs (4-7) as a subset. Now we must discuss the nature of the branes of extension 1, 6 and 9. For the first two cases we have obvious candidates. The $M1$-brane can be interpreted as the Brinkmann wave in $D = 11$:

$$ds^2 = (2 - H)dt^2 - Hdz^2 + 2(1 - H)\, dtdz - (dx_2^2 + ... + dx_{10}^2), \tag{13}$$

where H is a harmonic function in the variables $t + z, x_2, \ldots, x_{10}$. Its interpretation as an $M1$-brane makes sense, since it indeed preserves 1/2 supersymmetry, and its direct dimensional

reduction to $D = 10$ gives the fundamental string solution. The double dimensional reduction gives the $D0$-brane in $D = 10$.

Also the $M6$-brane allows a natural interpretation. It must be the Kaluza-Klein monopole [11], with metric ($i = 1, 2, 3$)

$$ds^2 = dt^2 - dx_1^2 - \ldots - dx_6^2 - H^{-1}(dz + A_i dy_i)^2 - H dy_i^2 \,, \tag{14}$$

where H and A_i depend on y_i, and the relation between H and A_i is

$$F_{ij} \equiv \partial_i A_j - \partial_j A_i = \epsilon_{ijk} \partial_k H \,. \tag{15}$$

Direct dimensional reduction to $D = 10$ gives a $D6$-brane, double dimensional reduction the solitonic fivebrane in $D = 10$. Recently we have extended our results on $M2$- and $M5$-branes [8] to include also the wave (13) and the monopole (14) [12]. Interestingly, the intersections of pairs of waves and monopoles with $M2$ and $M5$, and with themselves, are precisely as given in Table 1. This, and the results on multiple intersections [8], gives us some confidence that supersymmetry may indeed be used to predict the allowed configurations of intersecting branes. According to this point of view, the construction of a multiple intersections involving N basic objects is the same as the construction of N commuting matrices $\Gamma^{0(p_i)}$, $i = 1, \ldots N$, where (p_i) denotes the spatial orientation of the worldvolume of the p_i-brane.

There is no known 9-brane solution of $D = 11$ supergravity. Nevertheless, the above results indicate that we should seriously consider the existence of such an object[5]. There are also other indications that a 9-brane should exist. In $D = 10$ there is an $D8$-brane solution [18, 13], and, according to the M-theory interpretation of string theories, it should have an eleven-dimensional counterpart. However, the $D8$-brane requires the massive extension of $D = 10$ IIA supergravity [19], which we do not know how to lift to $D = 11$.

Our analysis does not tell us what the conjectured 9-brane solution is. But, assuming that it preserves 1/2 supersymmetry, and that the condition of preservation of supersymmetry is of the standard form, its pair intersections with the known solutions of $D = 11$ supergravity are determined (see Table 1). For instance, this analysis tells us that the 9-brane can occur in configurations of n $M5$-branes for $n \leq 7$. Such configurations would reduce in $D = 10$ to an intersection of n $D4$-branes with the $D8$-brane, which is known to be a solution of massive $D = 10$ IIA supergravity.

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